# DRAG AND HEAT TRANSFER IN LAMINAR MOTION OF INCOMPRESSIBLE FLUID WITH VARIABLE PHYSICAL PROPERTIES IN A TUBE WITH POROUS WALLS IN THE QUASIDEVELOPED-FLOW REGION

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The local-similarity method is used to find numerical solutions of the equations of motion and energy in a circular tube with blowing.

Investigation of the heat transfer and drag in laminar flow in tubes with permeable walls, taking variability of the physical properties into account, has attracted attention, as usual, in connection with its wide practical applications in various fields of modern engineering, and a large number of publications have now been devoted to this subject. The motion of a fluid with variable physical properties in a tube with permeable walls was considered in [1, 2], in which only the results calculated for the thermal characteristics of the flow were given.

The known methods of theoretical solution of these problems may be divided into two groups. The first group consists of methods based on approximate separation of variables and may be used to calculate the flow in regions of hydrodynamic and thermal quasistabilization; the most systematic application of this approach is to be found in [3]. The other groups collects together works (e.g., [4-7, 1]) in which partial differential equations are solved by the finite-difference method. This solution yields more complete information on the flow including the inlet region; however, such calculations are considerably more laborious than obtaining a solution for quasideveloped flow, and therefore its use in engineering practice may lead to definite difficulties.

The main drawback of the approach based on variable separation is, as noted in [8, 9], the considerable error in determining the frictional drag. This is because of the complete neglect of convective terms in the equation of motion; the most important of these, according to [8], is the transverse convection associated with the appearance of a radial velocity induced by the variation in density. The inclusion of the radial velocity in a one-dimensional calculation scheme, as in [8], is a fairly artificial method, and its importance may only be determined empirically, by comparison with experimental data. In [10], it was suggested that a method taking the flow history and convective transfer in the axial direction into account may be developed for the quasideveloped-flow region. On this basis, the present work attempts to develop the calculation scheme of [3] so as to take account of dynamic effects in the axial direction, which are no less important than the effects of radial convection and may be taken more systematically into account in the framework of a one-dimensional method. The method of solution adopted is also used to solve the equations of energy and motion for blowing through porous walls.

Consider steady axisymmetric motion in a circular tube with boundary conditions of the second kind and a blowing velocity at the wall constant over the length. The system of equations describing mass, momentum, and heat transfer in the boundary-layer-theory approximation, disregarding energy dissipation, takes the form

$$\frac{\partial(\rho u_x)}{\partial x}+\frac{1}{r}\frac{\partial(r\rho u_r)}{\partial r}=0, \tag{1}$$

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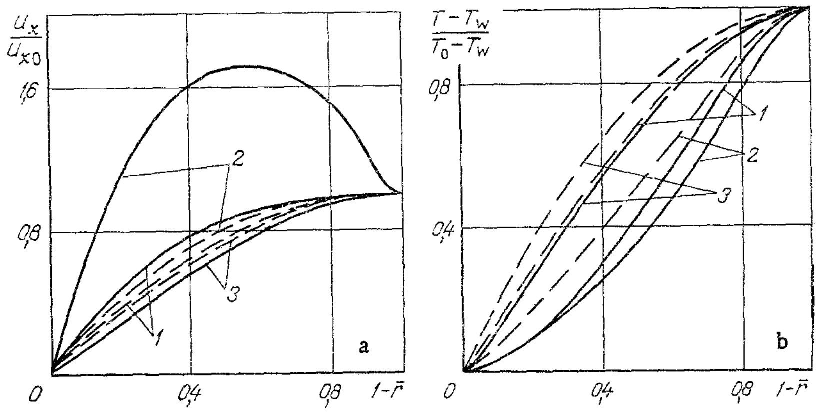


Fig. 1. Velocity (a) and temperature (b) distribution over tube cross section; the continuous curves correspond to $Re_V = -15$ and the dashed curves to $Re_V = 0$; $\theta = 1$ (1), 1.35 (2), 0.5 (3).

$$\rho u_x \frac{\partial u_x}{\partial x} + \rho u_r \frac{\partial u_r}{\partial r} = -\frac{dp}{dx} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u_x}{\partial r}\right), \tag{2}$$

$$\rho u_x \frac{\partial h}{\partial x} + \rho u_r \frac{\partial h}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda\frac{\partial T}{\partial r}\right). \tag{3}$$

Integrating the continuity equation — Eq. (1) — over the tube cross section yields the variation in mean velocity over the length, which, according to the definition $U_m = \overline{\rho w}/\rho_m$, is

$$\frac{dU_m}{dx} = -\frac{2\rho_w V_W}{\rho_m r_0} - \frac{U_m}{\rho_m}\frac{d\rho_m}{dx}. \tag{4}$$

The first term in Eq. (4) describes the variation in mean velocity due to blowing, and the second the variation due to the variable density produced by heating or cooling of the liquid.

Taking account of Eq. (1), Eq. (3) yields the heat-balance equation

$$\frac{dh_m}{dx} = \frac{2}{r_0\overline{\rho w}}\left[\lambda_W\left(\frac{\partial T}{\partial r}\right)_{r=r_0} + \rho_W V_W(h_m - h_W)\right] = \frac{2q_W}{r_0\overline{\rho w}}. \tag{5}$$

In Eq. (5) it is assumed that, in the presence of blowing, the total heat-flux density through the wall due to heat conduction and convection is known.

Neglecting the pressure dependence of the physical properties in comparison with the temperature dependence, which does not lead to appreciable error at subsonic flow velocities and in the region of state parameters far from the saturation curve, Eq. (4) is written in the form

$$\frac{dU_m}{dx} = -\frac{2\rho_W V_W}{\rho_m r_0} - \frac{2q_w}{r_0 c_{pm}\rho_m}\left(\frac{\partial\rho}{\partial T}\right)_m. \tag{6}$$

Below, the flow of a perfect gas will be considered, assuming the Clapeyron–Mendeleev equation of state

$$p = R\rho T. \tag{7}$$

Using Eqs. (5)-(7), assuming that the derivative $\partial h/\partial x$ is constant over the tube cross section, and adopting the local-similarity hypothesis, which is widely used in calculations of quasideveloped flow, i.e., introducing a relation of the form $u_x/U_m = f(\bar{r})$, solution of the system of partial differential equations in Eqs. (1)-(3) may be approximately reduced to solving the dimensionless ordinary differential equations

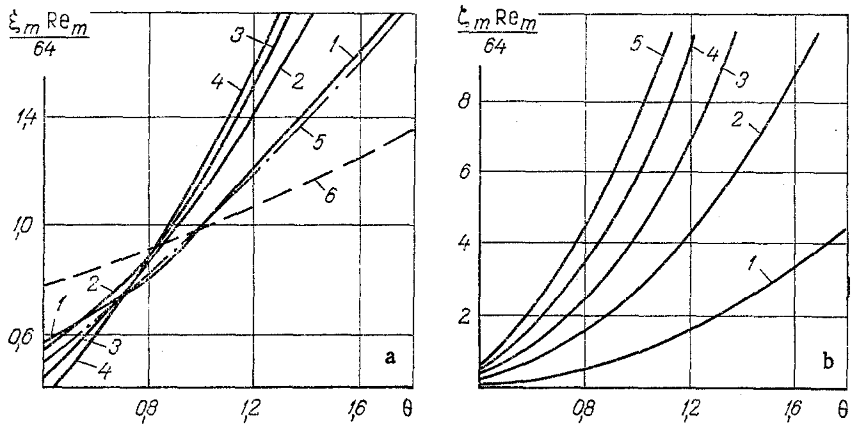


Fig. 2. Dependence of frictional drag (a) — $Re_V = 0$ (1, 5, 6), 5 (2), 10 (3), and 20 (4) — and hydraulic drag (b) — $Re_V = 0$ (1), 5 (2), 10 (3), 15 (4), and 20 (5) — on temperature factor.

$$\frac{1}{\bar r}\frac{d}{d\bar r}(\bar r\bar\rho\bar u_r) - 2\bar\rho\bar u_x = 0, \tag{8}$$

$$\frac{1}{\bar r}\frac{d}{d\bar r}\left(\bar r\bar\mu\frac{d\bar u_x}{d\bar r}\right) - \frac{Re_V}{2}\bar\rho\bar u_r + (Re_V - Q_W\theta N_c)\bar\rho\bar u_x^2 + K = 0, \tag{9}$$

$$\frac{1}{\bar r}\frac{d}{d\bar r}\left(\bar r\bar\lambda\frac{d\bar T}{d\bar r}\right) - \frac{Re_V Pr_W}{2}\bar\rho\bar u_r\bar c_p\frac{d\bar T}{d\bar r} - Q_W Pr_W\bar u_x\bar\rho = 0. \tag{10}$$

Here

$$\bar r = \frac{r}{r_0};\quad \bar u_x = \frac{u_x\rho_W}{\overline{\rho w}};\quad \bar u_r = \frac{u_r}{V_W};\quad \bar T = \frac{T}{T_W};\quad \bar\rho = \frac{\rho}{\rho_W};\quad \bar\mu = \frac{\mu}{\mu_W};$$

$$\bar\lambda = \frac{\lambda}{\lambda_W};\quad \bar c_p = \frac{c_p}{c_{pW}};\quad N_c = \frac{c_{pW}}{c_{pm}};\quad K = -\frac{r_0^2\rho_W^2}{\overline{\rho w}\rho_m\mu_W}\frac{dp}{dx}.$$

In the absence of blowing ($Re_V = 0$), the only significant difference between the present method of calculation and that of [3] is that explicit account is taken of the convective term in Eq. (9) describing flow acceleration on heating (deceleration on cooling); in [3] this term is combined with the pressure gradient. Estimates show that separation of the convective term is expedient at high heat fluxes, when inertial terms are found to be comparable with viscosity terms as a result of the density variation.

The system in Eqs. (8)-(10) was solved by the trial-and-error method, with iterations for the following boundary conditions

$$\bar r = 0\ \bar u_r = \frac{\partial\bar u_x}{\partial\bar r} = \frac{\partial\bar T}{\partial\bar r} = 0;\quad \bar r = 1\ \bar u_x = 0,\ \bar T = 1.$$

The parameters adopted are the dimensionless heat flux $Q_W$ and the Reynolds number $R_V$ for the flux at the wall. The dimensionless pressure gradient is determined using the condition that $\bar u_r = 1$ when $\bar r = 1$ or, according to Eq. (8), $\int_0^1 \bar r\bar\rho\bar u_x d\bar r = 1/2$. . Calculations are made for air, the properties of which are determined from the approximate formulas of [6].

The results obtained for the velocity and temperature distribution over the tube cross section are shown in Fig. 1. It is evident from Fig. 1a that the effect of variable physical properties on the velocity distribution is much stronger in the case of blowing than for constant flow rate. However, in contrast to [3], the velocity profile for $Re_V = 0$ becomes fuller in the case of heating, and somewhat less full on cooling. An interesting feature of the velocity distribution with blowing and heating is the shift of the maximum of $u_x$ from the tube axis to the wall with increase in $Re_V$ and θ. This is because increase in blowing intensity is associated with rise in the negative pressure gradient responsible for the acceleration of the light gas in the near-wall region. An analogous effect was observed experimentally for a permeable plate in [11] with injection of a lower-density gas (helium) into an air flow. The effect of variable physical properties on the temperature distribution in tubes with impermeable and permeable walls is qualitatively the same, and consists in reduction in fullness of the profile on heating and increase on cooling.

In Fig. 2, results obtained for the frictional drag and hydraulic drag are shown. The dashed curve (curve 6) shows the dependence for the frictional drag at $Re_V = 0$ obtained from a calculation disregarding thermal acceleration or deceleration of the flow, which is equivalent to the method of [3]. As is evident from Fig. 2, taking axial convection into account in the equation of motion has a significant effect on the dynamic characteristics of the flow. Acceleration of the flow on heating leads to a marked increase in both the frictional drag and the total hydraulic drag. The dependence of the frictional coefficient on the temperature factor obtained when acceleration (deceleration) of the flow is taken into account (curve 1) is in considerably better agreement with the results of two-dimensional solution [4] (curve 5) and experimental data [8] than the corresponding dependence neglecting axial convection. On cooling, the recovery in pressure head due to slowing of the flux leads to sharp decrease in the hydraulic drag. In the case of blowing, as for $Re_V = 0$, heating leads to considerable increase, and cooling to decrease, in the friction and hydraulic drag.

Calculations show that the blowing velocity in the range $-20 \leq Re_V \leq 0$, the effect of variable physical properties on the Nusselt number is slight. Therefore, results obtained for constant physical properties may be used to calculate the heat transfer.

## NOTATION

$u_x$, $u_r$, axial and radial velocity components; p, pressure; h, enthalpy; T, temperature; $\rho$, density; $\mu$, **dynamic viscosity**; $\lambda$, heat conduction; $c_p$, specific heat: $r_0$. tube radius; $q_W$, heat-flux density at wall; $\tau_w$, wall shear stress; $v_W$, blowing velocity ($V_W < 0$)$\cdot$ $\theta = T_W/T_m$, temperature factor; $Pr_W = \mu_W c_p W$; $\overline{\rho w} = 2\int_0^{r_0} \rho u_x r dr/r_0^2$, mass velocity; $h_m = 2\int_0^{r_0} h\rho u_x r dr/r_0^2 \overline{\rho w}$, mean-mass enthalpy; $T_m$; temperature corresponding to mean enthalpy; $Re_m = 2\rho\bar{w}\bar{r}_0/\mu_m$, axial Reynolds number; $Re_V = 2V_W\rho_W/\mu_W$, radial Reynolds number; $Q_W = 2q_W r_0 T_W c_{pW}\mu_W$, dimensionless heat flux at wall; $\xi_m = 8\tau_W\rho_m/\rho\bar{w}^2$, frictional-drag coefficient; $\zeta_m = -(4r_0\rho_m/\rho_W^2)dp/dx$, hydraulig-drag coefficient) R, universal gas constant. Indices: W, m, values determined at wall temperature $T_W$ and mean-mass temperature $T_m$.

## LITERATURE CITED


1. J. R. Doughty and H. C. Perkins, "Variable properties of laminar-gas-flow heat transfer in the entry region of parallel porous plates," Int. J. Heat Mass Transfer, 16, No. 3, 663-666 (1973).
2. S. K. Vinokurov, "Effect of change in physical properties on heat transfer in porous tube," in: Intensifying Energy and Mass Transfer in Porous Media at Low Temperatures, ITMO im. A. I. Lykov, Minsk (1975), pp. 111-116.
3. B. S. Petukhov and V. N. Popov, "Theoretical calculation of heat transfer and frictional drag for laminar flow in tubes of incompressible fluid with variable physical properties," Teplofiz. Vys. Temp., 1, No. 2, 228-237 (1963).
4. P. M. Worsfe-Schmidt and G. Leppert, "Heat transfer and friction for laminar flow of gas in a circular tube at high heating rate. Solutions for hydrodynamically developed flow by a finite-difference method," Int. J. Heat Mass Transfer, 8, No. 10, 1281-1301 (1965).
5. V. O. Vilenskii, B. S. Petukhov, and B. E. Kharin, "Heat transfer and drag in circular tube for laminar flow of gas with variable properties. 1. Method of Calculation," Teplofiz. Vys. Temp., 7, No. 5, 931-939 (1969).
6. V. D. Vilenskii, B. S. Petukhov, and B. E. Kharin, "Heat transfer and drag in circular tube for laminar flow of gas with variable properties. 2. Results of calculation at constant wall temperature," Teplofiz. Vys. Temp., 8, No. 4, 817-827 (1970).
7. V. D. Vilenskii, B. S. Petukhov, and B. E. Kharin, "Heat transfer and drag in circular tube for laminar flow of gas with variable properties. 3. Results of calculations at constant heat-flow density at wall," Teplofiz. Vys. Temp., 9, No. 3, 563-570 (1971).

8. Davenport and Leppert, "Effect of transverse temperature gradients in heat transfer and friction in laminar gas flow," Teploperedacha, 87, Ser. C, No. 2, 36-43 (1965).
9. B. S. Petukhov, Heat Transfer and Drag in Laminar Fluid Flow in Tubes [in Russian], Énergiya, Moscow (1967).
10. Makéligot, Smit, and Benkston, "Quasideveloped turbulent flow in a tube in the presence of heat transfer," Teploperedacha, 92, Ser. C, No. 4, 67-79 (1970).
11. V. M. Eroshenko, A. L. Ermakov, A. A. Klimov, V. P. Motulevich, and Yu. N. Terent'ev, "Investigation of laminar boundary layer at permeable surface," in: Heat and Mass Transfer [in Russian], Vol. 1, Part 1, Minsk (1972), pp. 176-180.

# EFFICIENCY OF POROUS COOLING

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The authors present results of a numerical analysis of the efficiency of porous cooling of a cylindrical tube with one-sided heating, accounting for the thermophysical properties being temperature dependent.

The efficiency of transpiration or porous cooling of structural elements is determined by the thermophysical and hydraulic characteristics of the porous material and by the type and mass flow rate of the coolant [1]. A well-founded choice of material and coolant ensures operational capability of thermally stressed structural elements, e.g., the arc-stabilizing porous tubes of interelectrode inserts (IEI) of plasmatrons, at the optimal coolant flow rates.

The results of investigations of processes of heat transfer and hydrodynamics in porous media have been correlated in [2, 3]. The main studies have been a filtration regimes in conditions with comparatively small temperature drop, up to 100°K through the wall thickness. The specific mass flow rate of gas through the porous wall of the IEI tube reaches 500 kg/m²·sec (air), and the heat flux to the wall reaches $10^8$ W/m². In these conditions the temperatures of the material and the coolant vary through the wall thickness from the temperature of the gas or liquid at the entrance to the test facility, up to the limiting working temperature of the material, e.g., from liquid nitrogen temperature (77°K) to the limiting working temperature of tungsten (2900°K) [4]. It becomes particularly important in this case to allow for the dependence of the properties of the material and the coolant on temperature in resolving the thermal and hydraulic problems of porous cooling.

The system of equations for heat transfer and hydrodynamics in a porous nondeformable medium, neglecting viscous dissipation and the kinetic energy of the gas relative to the thermal, and assuming an optically thin layer of coolant, in the one-dimensional approximation for a cylinder, has the form (the computational scheme is shown in Fig. 1):

$$\frac{1}{r}\frac{d}{dr}\lambda_{\text{eff}}\frac{dT_w}{dr}-\alpha_v(T_w-T_g)=0, \tag{1}$$

$$c_p m\frac{dT_g}{dr}=\alpha_v(T_w-T_g),\quad -\frac{dP}{dr}=\alpha\mu\frac{m}{\rho}+\beta\frac{m^2}{\rho}, \tag{2}$$

$$\rho=\frac{P}{RT_g},\qquad m=\frac{G}{2\pi r}. \tag{3}$$

As the basic dependence of $\alpha_v$ on m we take

$$\alpha_v=0.029\,\mathrm{Re}^{1.84}\lambda_g/(\beta/\alpha)^2.$$

The boundary conditions are as follows:

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